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# DIFFRACTION OF A PLANE ELECTROMAGNETIC WAVE ON A POORLY CONDUCTING, CELLULAR STRUCTURE IN A DIELECTRIC LAYER

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Various ways are known to control the reflectivity of surfaces, both to reduce energy losses in the reflection of an electromagnetic wave from a surface with a finite conductivity and to increase those losses in order to reduce the intensity of the reflected waves. The first of these problems is solved, in particular, using a comb comprised of strips of a well-conducting material. There are a review of the corresponding experimental data and a theoretical confirmation of them in [1]. Experimental research [2] has shown that a similar comb made of poorly conducting strips enables one to solve the opposite problem of reducing the intensity of the reflected wave; the well-known resonant absorber in the form of a thin, poorly conducting layer located at a distance $\lambda/4$ in front of the conducting base works in the same way.

It is of interest to investigate the diffraction of an electromagnetic wave on a periodic structure that is the result of combining an absorbing comb and a thin absorbing layer, to clarify the possibilities for controlling the absorptivity of the resulting structure.

The present paper is devoted to a theoretical solution of this problem. The geometry of the problem is given in Fig. 1 in the form of a cross section of the structure under consideration, which consists of rectangular cells resting on a plane with a conductivity $\sigma = \infty$. The cells are formed by thin, poorly conducting walls, and the space inside the cells is filled with a dielectric having a dielectric constant $\varepsilon$. The arrangement of the coordinate system and all the geometrical characteristics are shown in Fig. 1. Note that by the term "thin, poorly conducting wall" we mean a plane layer of a conducting (with a conductivity $\sigma_1$) material whose thickness $d_1$ is considerably less than either the wavelength or the thickness of the skin layer in the wall material, i.e.,

$$d_1 \ll \frac{\lambda_0}{\sqrt{\varepsilon}}, \qquad d_1 \ll \frac{c}{\sqrt{2\pi\sigma_1\omega}}. \tag{1}$$

The thickness of this layer therefore cannot enter into the problem as a physical parameter; the only parameter characterizing the layer is its surface conductivity $\Lambda = \sigma_1 d_1$, which determines the surface density of current in the layer to be $i_z = \Lambda E_z$. The surface conductivity $\Lambda$ of the vertical walls comprising the comb and the conductivity $\Lambda_1$ of the horizontal layer lying in the plane $y = 0$ are assumed to differ, in general (Fig. 1).

The incident wave is linearly polarized with $E_z \neq 0$ and the wave vector $\mathbf{k}_0$ lies in the $(x, y)$ plane. Using the symbol $\varphi$ with the appropriate indices to designate the amplitude of $E_z$ [the time factor is taken in the form $\exp(-i\omega t)$], we take the indicent wave in the form

$$\varphi_0 = \exp(-ik_0 y\cos\theta_0 + ik_0 x\sin\theta_0).$$

The field in the half-space $y > 0$ will then be characterized by the sum $\varphi_0 + \varphi_1(x, y)$, where $\varphi_1$ is the amplitude of the reflected wave, while we mark the field in the strip $-b < y < 0$ by the index 2.

The fields being sought satisfy the equations

$$\Delta\varphi_i + k_i^2\varphi_i = 0, \quad k_i = \begin{cases} k_0 & \text{for } i = 1, \\ \sqrt{\varepsilon}k_0 & \text{for } i = 2 \end{cases}$$

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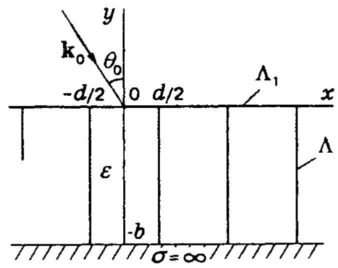


Fig. 1

and the appropriate boundary conditions: the condition of emission as $y \to \infty$, the condition

$$\varphi_2(x, -b) = 0 \tag{2}$$

at the superconducting substrate, and the conditions at the poorly conducting walls that form the cells of the given structure. By virtue of (1), the latter come down to continuity of the field $E_z$ and the presence of discontinuities of the tangential components of the field $\mathbf{H}$ at those walls. For the boundary $y = 0$, in particular, these conditions are

$$\begin{gathered}\varphi_2(x,0) - \varphi_1(x,0) = \exp(ik_0 x \sin\theta_0)\,,\\ \frac{\partial\varphi_2}{\partial y}(x,0) - \frac{\partial\varphi_1}{\partial y}(x,0) - i\frac{4\pi\omega\Lambda_1}{c^2}\varphi_2(x,0) = -ik_0\cos\theta_0\exp(ik_0 x\sin\theta_0)\,.\end{gathered} \tag{3}$$

Hence it follows that as the $x$ coordinate changes by a multiple number of periods $d$ of the structure, the solutions being sought must acquire an additional phase

$$\varphi_i(x + nd, y) = \varphi_i(x,y)\exp(inu), \qquad u = k_0 d\sin\theta_0\,. \tag{4}$$

With allowance for this fact, the remaining boundary conditions at the comb walls can be written as equations connecting $\varphi_2$ and $\partial\varphi_2/\partial x$ at the right and left sides of a given cell. Using the internal coordinate $x'$, where $x = nd + x'$, $n = 0, \pm 1, \pm 2, \ldots$, we have

$$\begin{gathered}\varphi_2\Big|_{x'=d/2} = \varphi_2\Big|_{x'=-d/2}\exp(iu)\,,\\ \frac{\partial\varphi_2}{\partial x'}\Big|_{x'=d/2} - \frac{\partial\varphi_2}{\partial x'}\Big|_{x'=-d/2}\exp(iu) = i\frac{4\pi\Lambda\omega}{c^2}\varphi_2\Big|_{x'=d/2}\,.\end{gathered} \tag{5}$$

The solution for the region $-b < y < 0$ that satisfies the conditions (2) and (4) can be represented as

$$\begin{aligned}\varphi_2(x,y) = \sum_{m=1}^{\infty}&(A_m\sin\alpha_m x' + B_m\cos\alpha_m x')\times\\ &\times\left[\beta_m\exp(ik_m^\varepsilon y) - \left(\frac{1}{\beta_m}\right)\exp(-ik_m^\varepsilon y)\right]\exp(inu)\,,\end{aligned} \tag{6}$$

where

$$k_m^\varepsilon = \sqrt{k_0^2\varepsilon - \alpha_m^2} \quad (\operatorname{Re}k_m^\varepsilon \geqslant 0, \quad \operatorname{Im}k_m^\varepsilon \geqslant 0), \quad \beta_m = \exp(ik_m^\varepsilon b)\,. \tag{7}$$

The constants $A_m$ and $B_m$ and the eigenvalues $\alpha_m$ are determined from the boundary conditions (5), from which we get a homogeneous system of equations that must have a nontrivial solution:

$$\begin{gathered}A_m\sin\frac{z_m}{2}(1+\exp(iu)) = B_m\cos\frac{z_m}{2}(\exp(iu)-1)\,,\\ A_m\frac{z_m}{2}\cos\frac{z_m}{2}(1-\exp(iu)) =\end{gathered} \tag{8}$$

$$= B_m \frac{z_m}{2} \sin \frac{z_m}{2} (\exp(iu) + 1) + i\eta (A_m \sin \frac{z_m}{2} + B_m \cos \frac{z_m}{2}) .$$

Here

$$z_m = \alpha_m d; \quad \eta = \frac{4\pi\Lambda}{c} \frac{k_0 d}{2} .$$

Let us consider two particular cases.

1. $\exp(iu) = 1$ or $u = k_0 d \sin(\theta_0) = 2n\pi$. The case of direct incidence, $u = 0$, does not come in here. Then from $A_m \sin(z_m/2) = 0$ we get either

$$A_m = 0, \qquad z_m \sin(z_m/2) + i\eta \cos(z_m/2) = 0 ,$$

or

$$\sin(z_m/2) = 0, \qquad B_m = 0 .$$

Finally, the general solution has the form

$$\varphi_2(x,y) = \sum_{m=1}^{\infty} \{A_m \sin(2m\pi x'/d)[\exp(ik^{\varepsilon}_{m1}y)\beta_{m1} - \exp(-ik^{\varepsilon}_{m1}y)/\beta_{m1}] + \\ + B_m \cos(\alpha_m x')[\exp(ik^{\varepsilon}_{m2}y)\beta_{m2} - \exp(-ik^{\varepsilon}_{m2}y)/\beta_{m2}]\} \exp(inu) , \tag{9}$$

where

$$k^{\varepsilon}_{m1} = \sqrt{k_0^2 \varepsilon - (2m\pi/d)^2}, \quad k^{\varepsilon}_{m2} = \sqrt{k_0^2\varepsilon - \alpha_m^2}, \quad \alpha_m = z_m/d , \\ \beta_{m1} = \exp(ik^{\varepsilon}_{m1}b) , \quad \beta_{m2} = \exp(ik^{\varepsilon}_{m2}b) , \quad (z_m/2)\mathrm{tg}(z_m/2) = -i\eta/2 . \tag{10}$$

2. $\exp(iu) = -1$ or $u = k_0 d \sin(\theta_0) = (2n - 1)\pi$. By analogy with case 1, we write the general solution in the form

$$\varphi_2(x,y) = \sum_{m=1}^{\infty} \{A_m \sin(\alpha_m x')[\exp(ik^{\varepsilon}_{m2}y)\beta_{m2} - \exp(-ik^{\varepsilon}_{m2}y)/\beta_{m2}] + \\ + B_m \cos((2m-1)\pi x'/d)[\exp(ik^{\varepsilon}_{m1}y)\beta_{m1} - \exp(-ik^{\varepsilon}_{m1}y)/\beta_{m1}]\} \exp(inu). \tag{11}$$

Here

$$k^{\varepsilon}_{m1} = \sqrt{k_0^2\varepsilon - [(2m-1)\pi/d]^2}, \quad k^{\varepsilon}_{m2} = \sqrt{k_0^2\varepsilon - \alpha_m^2}, \quad \alpha_m = z_m/d , \\ \beta_{m1} = \exp(ik^{\varepsilon}_{m1}b) , \quad \beta_{m2} = \exp(ik^{\varepsilon}_{m2}b) , \quad (z_m/2)\mathrm{ctg}(z_m/2) = i\eta/2 . \tag{12}$$

In the general case $\exp(iu) \neq \pm 1$, we write the spectral equation for the eigenvalues $z_m = \alpha_m d$ of the problem (8) as

$$(z_m/2)[\mathrm{ctg}(z_m/2)\sin^2(u/2) - \mathrm{tg}(z_m/2)\cos^2(u/2)] = i\eta/2 , \tag{13}$$

while we write the solution (6), (7) within cells of the structure as

$$\varphi_2(x,y) = \Big[\sum_{m=1}^{\infty} C_m h_m(x')(\exp(ik^{\varepsilon}_m y)\beta_m - \exp(-ik^{\varepsilon}_m y)/\beta_m)\Big] \exp(inu) , \tag{14}$$

where $k_m^\varepsilon = \sqrt{k_0^2\varepsilon - \alpha_m^2}$; $\alpha_m = z_m/d$, while the eigenfunctions are

$$h_m(x') = \cos(\alpha_m x') + i\mathrm{tg}(u/2)\mathrm{ctg}(z_m/2)\sin(\alpha_m x') \,. \tag{15}$$

The validity of such a representation requires some additional justification (see, e.g., [3, 4]). Strictly speaking, a system of eigenfunctions of a non-self-adjoint operator in Hilbert space [including the operator that implements a Helmholtz equation with the boundary conditions (5)] is incomplete, since a complete system also includes the associated functions generated when there are multiple eigenvalues. It was shown in [1] that for Eqs. (10) and (12) under the condition $\mathrm{Im}(\eta) \geq 0$, including real $\eta$, the multiplicity of roots does not exceed one. Using a similar approach, we can show that the spectral equation (13) for real $\eta$ cannot have multiple roots, which will mean that the representation (14) is valid.

Note that if $z_m$ is a root of Eq. (13), then $-z_m$ is also a root and it generates the same eigenfunction (15). We can thus obtain all of eigenfunction space by confining the analysis to one half-plane [for determinacy, $\mathrm{Re}(z_m) \geq 0$].

Following [4], solutions (13) for a zero right side, $\eta = 0$, are called exit points for $z_m$, and solutions for $\eta \to +\infty$ are called entrance points. It is obvious that $z^{\mathrm{ex}} = \pm u + 2l\pi$ and $z^{\mathrm{en}} = n\pi$. Exit and entrance points lie on the real axis and alternate, i.e., after an exit point comes an entrance point, then another exit point, etc. We number the exit points (and hence the solutions for $\eta > 0$) starting with the smallest positive value of $z^{\mathrm{ex}}$ in the direction of an increase in the real part. The dynamics of the values of $z_m$ with variation of $\eta$ from zero can be determined from an analysis of Eq. (13) in the vicinity of an exit point. As a result, we have $z_m \approx z_m^{\mathrm{ex}} - i\eta/z_m^{\mathrm{ex}}$, i.e., for $z_m^{\mathrm{ex}} > 0$, the curves on which roots $z_m$ are located emerge into the lower half of the complex plane. Since there are no solutions on the real axis but entrance and exit points (except for the case of $u = n\pi$, which is mentioned below), the set of solutions $\{z_m\}$ lies entirely in quadrant IV of the complex plane. The correspondence of entrance and exit points is established by analogy with the allowance for second-order terms, as well as an analysis of the differential equation

$$\frac{dz_m}{d\eta} = \frac{i\sin z_m}{\cos z_m - \cos u - z_m \sin z_m - i\eta\cos z_m} \tag{16}$$

that follows from (13), with the initial conditions $z_m|_{\eta=0} = z_m^{\mathrm{ex}}$.

A numerical solution of (16) showed that the roots actually lie in quadrant IV and are located on curves that emerge from the points $z_m^{\mathrm{ex}}$ and arrive at the entrance point closest to the exit point and lying to the right (for $z_m^{\mathrm{ex}} > 0$). If $u = n\pi$, then some of the entrance points coincide with exit points and the roots corresponding to them lie on the real axis [which is taken into account in Eqs. (9) and (11)].

We now show that the multiplicity of roots is one everywhere. The proof is carried out from the contrary. Assume that some root $z_m$ has a multiplicity of at least two. Then along with Eq. (13), which it is convenient to rewrite in the form

$$f(z) = z(\cos z - \cos u) - i\eta\sin z = 0\,, \tag{17}$$

the following equation must be satisfied:

$$f'(z) = (\cos z - \cos u) - z\sin z - i\eta\cos z = 0\,. \tag{18}$$

We rewrite Eqs. (17) and (18) as

$$\begin{gathered}\exp(2iz)(z-\eta) - 2z\cos u\exp(iz) + (z+\eta) = 0\,,\\ \exp(2iz)(z-\eta-i) + 2i\cos u\exp(iz) - (z+\eta+i) = 0\,.\end{gathered} \tag{19}$$

Eliminating $\exp(iz)$ from (19), we obtain a biquadratic equation for $z$:

$$(z^2-\eta^2-i\eta)^2 = \beta^2((z^2-i\eta)^2 - \eta^2 z^2)\,, \quad \beta = \cos u\,, \quad \beta^2 < 1 \tag{20}$$

(the case of $\beta^2 = 1$ corresponds to the spectral equation (10) considered in [1]). Solving (20) for $y = z^2 - i\eta$, we obtain

$$y = \frac{(2-\beta^2)\eta^2/2 \pm \sqrt{D}}{1-\beta^2}$$

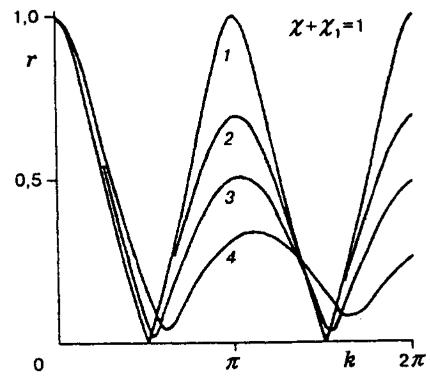


Fig. 2

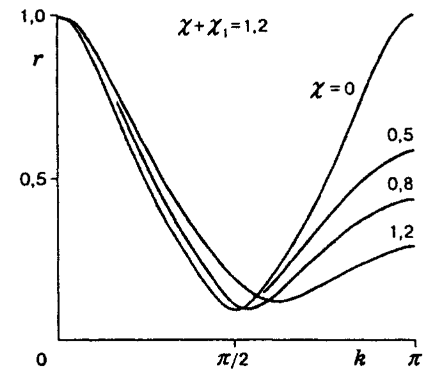


Fig. 3

$(D = \eta^2\beta^2(\eta^2\beta^2/4 - i\eta(1 - \beta^2)))$, from which we have for the quantity being sought

$$z^2 = i\eta + \frac{\eta^2(2-\beta^2)}{2(1-\beta^2)} \pm \frac{\beta\eta}{2(1-\beta^2)}\sqrt{\frac{\eta^2\beta^2}{2} + \sqrt{C}} \mp \mp \frac{i\beta\eta}{2(1-\beta^2)}\sqrt{-\frac{\eta^2\beta^2}{2} + \sqrt{C}} \tag{21}$$

$(C = \eta^4\beta^4/4 + 4\eta^2(1 - \beta^2)^2 > 0)$. It is easy to ascertain that both solutions (21) lie in the upper half-plane. In fact, the imaginary part is

$$\eta\left(1 \mp \frac{\beta}{2(1-\beta^2)}\sqrt{\frac{-\eta^2\beta^2}{2} + \sqrt{C}}\right) > 0\,,$$

since for $\beta^2 < 1$ and real $\eta$ and $\beta$, the following inequality is always valid:

$$4(1-\beta^2)^2 > \beta^2(\sqrt{\eta^4\beta^4/4 + 4\eta^2(1-\beta^2)^2} - \eta^2\beta^2/2)\,.$$

The solution $z^2$ thus lies in quadrant I or II of the complex plane, and then $z$ cannot lie in quadrant IV. This contradiction shows that Eqs. (17) and (18) are incompatible, and hence no multiple roots or associated functions exist, so the representation (14) for the solution inside a cell of the structure is justified.

The solution in the free space above the comb consists of the incident and diffracted electromagnetic waves. The latter can be represented in a form analogous to (14), with the only difference that here $\varepsilon = 1$ and $\Lambda = 0$, the eigenvalues coincide with exit points, and the $y$ dependence has a damped nature,

$$\varphi_1 = \Big[\sum_{m=1}^{\infty} D_m g_m(x') \exp(ik_m^0 y)\Big] \exp(inu)\,,$$

where the eigenfunctions are

$$g_m(x') = \cos(\gamma_m x') + i\mathrm{tg}(u/2)\mathrm{ctg}(t_m/2)\sin(\gamma_m x')\,; \quad t_m = z_m^{\mathrm{ex}}\,;$$

$$\gamma_m = t_m/d\,; \quad k_m^0 = \sqrt{k_0^2 - \gamma_m^2}\,; \quad \mathrm{Re}\,k_m^0 \geqslant 0\,; \quad \mathrm{Im}\,k_m^0 \geqslant 0\,.$$

The expansion coefficients $C_m$ and $D_m$ for the functions $\varphi_{1,2}$ are determined from the boundary conditions (3), from which we have

$$\exp(ik_0 \sin\theta_0 x') + \sum_{m=1}^{\infty} D_m g_m(x') = \sum_{m=1}^{\infty} C_m(\beta_m - 1/\beta_m)h_m(x')\,; \tag{22}$$

$$-k_0 \cos\theta_0 \exp(ik_0 \sin\theta_0 x') + \sum_{m=1}^{\infty} D_m k_m^0 g_m(x') = \sum_{m=1}^{\infty} C_m k_m^\varepsilon (\beta_m + 1/\beta_m) h_m(x') -$$

$$-\frac{4\pi}{c}\Lambda_1 k_0 [\, \exp(ik_0 \sin\theta_0 x') + \sum_{m=1}^{\infty} D_m g_m(x')] \,. \tag{23}$$

One way to change from (22) and (23) to an algebraic system of equations for $D_m$ and $C_m$ is to introduce the family of functions

$$p_m(x') = \cos(\alpha_m x') - i\, \mathrm{tg}(u/2) \mathrm{ctg}(z_m/2) \sin(\alpha_m x')$$

and the operator $I(f,g) = \int_{-d/2}^{d/2} f(x) g(x) dx$. Using the property

$$I(p_m, h_n) = \delta_{nm} I(p_m, h_m)$$

enables us to reduce the system (22), (23) to the form

$$\Big(-\cos\theta_0 + \frac{4\pi}{c}\Lambda_1\Big) I(g_0, p_n) + \sum_{m=1}^{\infty} D_m \Big(\frac{k_m^0}{k_0} + \frac{4\pi}{c}\Lambda_1\Big) I(g_m, p_n) =$$

$$= C_n \frac{k_n^\varepsilon}{k_0} \Big(\beta_n + \frac{1}{\beta_n}\Big) I(h_n, p_n) \,,$$

$$I(g_0, p_n) + \sum_{m=1}^{\infty} D_m I(g_m, p_n) = C_n \Big(\beta_n - \frac{1}{\beta_n}\Big) I(h_n, p_n) \,.$$

Eliminating $C_n$ from the equations, we obtain

$$\sum_{m=1}^{\infty} D_m \left\{ \frac{1}{\beta_n^2 - 1} \frac{k_n^\varepsilon}{k_0} - \frac{1}{\beta_n^2 + 1} \left( \frac{k_m^0}{k_0} + \frac{4\pi}{c}\Lambda_1 \right) \right\} I(g_m, p_n) =$$

$$= \left\{ \frac{1}{\beta_n^2 + 1} \left( -\cos\theta_0 + \frac{4\pi}{c}\Lambda_1 \right) - \frac{1}{\beta_n^2 - 1} \frac{k_n^\varepsilon}{k_0} \right\} I(g_0, p_n) \,. \tag{24}$$

where

$$I(g_m, p_n) = \frac{i\eta}{2} \frac{\cos(t_m/2)\cos(z_n/2)}{t_m^2 - z_n^2} d \,,$$

$$I(g_0, p_n) = \frac{i\eta}{2} \frac{\cos(u/2)\cos(z_n/2)}{u^2 - z_n^2} d \,. \tag{25}$$

In order to solve the algebraic system (24), (25) numerically, one must, first, confine oneself to a finite number of terms retained in the solution and, second, transform the conversion matrix. We accomplished the latter by multiplying each equation by $t_n^2 - z_n^2$, while the justification for the former was established from numerical experiments by comparing the results obtained for different numbers $N$ of retained terms. It turned out that to achieve $\sim 10^{-4}$ accuracy in calculating the structure's reflection and absorption coefficients, it is sufficient to confine ourselves to $N = 4$, and the main series of experiments was carried out for this $N$.

Some typical results of the calculations are represented graphically. It was found that the influence of the comb structure on the absorptivity of the subject under consideration is manifested optimally if the comb's period equals half the height of its component strips, i.e., for $d = b/2$. This geometry pertains to all of the graphs given here. The dielectric constant was taken everywhere to be $\varepsilon = 1.4$; the conductivities of the comb walls and of the dielectric coating deposited on top are characterized by the dimensionless values $\chi = 4\pi\Lambda/c$ and $\chi_1 = 4\pi\Lambda_1/c$, which are stipulated everywhere. The absolute dimensionless amplitude $r = |D_1|$ of the main reflected wave figures as the reflection coefficient in all cases, since for the parameters under consideration, such waves have amplitudes at least an order of magnitude smaller, and the fraction of the energy carried off by them can be neglected.

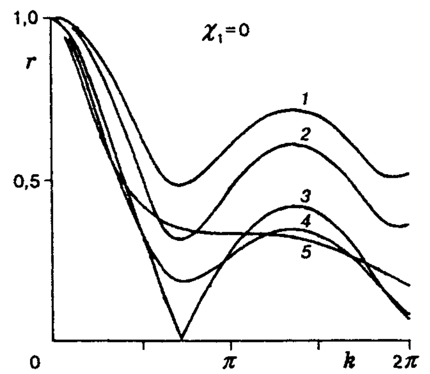


Fig. 4

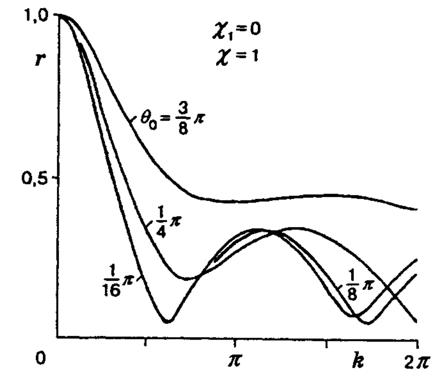


Fig. 5

In Fig. 2 we give the reflection coefficient as a function of the dimensionless wave number $k = k_0 b\sqrt{\varepsilon}$ for the case of direct incidence. The dimensionless conductivities $\chi$ and $\chi_1$ change from curve to curve, but their sum remains constant ($\chi + \chi_1 = 1$). Curve *1* pertains to a structure with no comb ($\chi = 0$), consisting of a dielectric layer and a deposited conductive coating with $\chi_1 = 1$; this structure is a resonant absorbing layer, and curve *1* is the well-known resonance curve $r(k)$, when the reflection coefficient is low in a narrow wavelength range while $r$ increases rapidly to unity with greater distance from resonance. Curves *2-4*, corresponding to $\chi = 0.3$, 0.6, and 1.0, allow for the presence of a comb and describe the dynamics of variation of the spectral absorption characteristic curve with variation of the structure's parameters. It is seen that the maximum value of $r$ between the two minima, which equals unity for a resonant layer, decreases with increasing $\chi$ and is 0.337 for $\chi = 1$. With a decrease in wavelength from resonance (short waves), a structure consisting of combs with no coating is preferable, while for wavelengths longer than resonance, a resonant absorbing layer continues to have some advantage.

Combining these two elements into one structure does not lead to broadening of the absorption spectrum. This fact is even more obvious for a high conductivity $\chi$. In this case, as seen from Fig. 3 ($\chi + \chi_1 = 1.2$, $\theta_0 = 0$), the first minima of $r$ for all of the curves do not reach zero, but in return, the maxima of $r$ that follow them are even lower for a pure comb than in Fig. 2, and the function $r(k)$ is even flatter. A structure consisting of a pure comb with $\chi > 1$ is thus consistent with a wide spread in $k$ while retaining a satisfactory absorption level.

The function $r(k)$ retains this character for an oblique incident wave. Here the reflection coefficient for a structure with no upper coating ($\chi_1 = 0$) equals zero for $\chi = \chi_* = \cos\theta_0$ and a layer thickness $b = \lambda/(4\cos\theta_0)$. In Fig. 4 we give the functions $r(k)$ for $\theta_0 = \pi/4$ for different $\chi$ [curves *1-5* correspond to $\chi = (1/3)\cos\theta_0$, $(1/2)\cos\theta_0$, $\cos\theta_0$, 1.0, and 1.5]. It is seen that the reflection coefficient is higher everywhere for $\chi < \chi_*$ than for $\chi = \chi_*$. The flattening of the function $r(k)$ noted above occurs with increasing $\chi$.

In Fig. 5 we give $r(k)$ for different angles of incidence and for a fixed $\chi = 1$. For $\theta_0 \leq \pi/8$ the curves differ little from the respective curve in Fig. 2, corresponding to normal incidence. The qualitative reorganization of this function begins at $\theta_0 > \pi/4$.

In conclusion, we note that, as shown by the data given here, combining a comb and a thin absorbing layer into one composite structure does not lead to the desired broadening of the absorption spectrum, as one would expect purely from theory.

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